Special Instructions / Useful Data

$\mathbb{Z}$ = The set of all integers
$\mathbb{Q}$ = The set of all rational numbers
$\mathbb{R}$ = The set of all real numbers
$\mathbb{C}$ = The set of all complex numbers
$\mathbb{R}^n = \mathbb{R} \times \mathbb{R} \times \cdots \times \mathbb{R}$, where $\mathbb{R}$ occurs n times
$\mathbb{C}^n = \mathbb{C} \times \mathbb{C} \times \cdots \times \mathbb{C}$, where $\mathbb{C}$ occurs n times
S_n = The symmetric group of all permutations on $1,2,3, \dots, n$
$\mathbb{Z}_n$ = The additive group of integers modulo n
$M_{m,n}(\mathbb{R})$ = The set of all $m \times n$ matrices with real entries
$M_{m,n}(\mathbb{C})$ = The set of all $m \times n$ matrices with complex entries
$M_n(\mathbb{R})$ = The set of all $n \times n$ matrices with real entries
$M_n(\mathbb{C})$ = The set of all $n \times n$ matrices with complex entries
M^T = The transpose of the matrix M
M^* = The conjugate transpose of the matrix M
$A \setminus B$ = The set of all elements of A which are not in B, for sets A and B
$[x]$ = The largest integer less than or equal to x for any real number x
$f'(x) = \frac{df}{dx}$
$f^{(k)}(x) = \frac{d^k f}{dx^k}$
$g \circ f$ is the composition function given by $(g \circ f)(x) = g(f(x))$

Section A: Q.1 – Q.10 Carry ONE mark each.	
Q.1	For each positive integer n, let $x_n = 1 - (-1)^n + \frac{1}{n}$ and $y_n = \left(1 + \frac{1}{2n}\right)^{3n}$. Which ONE of the following statements about the sequences (x_n) and (y_n) is TRUE?
(A)	(x_n) and (y_n) are convergent.
(B)	(x_n) is not convergent and (y_n) is not convergent.
(C)	(x_n) is convergent and (y_n) is not convergent.
(D)	(x_n) is not convergent and (y_n) is convergent.
Q.2	For each positive integer n, define $x_n = (-1)^n$. Which ONE of the following statements about the sequence (x_n) is FALSE?
(A)	There exists $\epsilon > 0$ such that $\lvert x_n - 1\rvert < \epsilon$ for all positive integers n.
(B)	There exists $\epsilon > 0$ such that for all $M > 0$ there exists a positive integer $N > M$ for which $\lvert x_N - 1\rvert > \epsilon$.
(C)	For all $\epsilon > 0$ and $M > 0$ there exists a positive integer N such that $N > M$ and $\lvert x_N - 1\rvert < \epsilon$.
(D)	For all $\epsilon > 0$ and $M > 0$ there exists a positive integer N such that $N > M$ and $\lvert x_N - 1\rvert > \epsilon$.

Q.3	Let $f: (1, 4) \to \mathbb{R}$ be a differentiable function such that $$f'(x) = (f(x) - \pi x)^2 + \pi \qquad \text{for all } x \in (1, 4).$$ Which ONE of the following is a possible value of $f(3) - f(2)$?
(A)	$\pi + \frac{1}{6}$
(B)	$\pi - \frac{1}{6}$
(C)	$\frac{\pi}{2} + \frac{1}{3}$
(D)	$\frac{\pi}{2} + \frac{1}{2}$

Q.4	The general solution of the differential equation $$x\,\sin\left(\frac{y}{x}\right)\frac{dy}{dx} = y\,\sin\left(\frac{y}{x}\right) + \frac{x}{2}, \quad x > 0$$ is given by ______.
(A)	$\cos\left(\frac{x}{y}\right) + \log_e(x^2) = K$, where K is an arbitrary constant
(B)	$\cos\left(\frac{y}{x}\right) + \log_e(x^2) = K$, where K is an arbitrary constant
(C)	$\cos\left(\frac{x}{y}\right) + \log_e(\sqrt{x}) = K$, where K is an arbitrary constant
(D)	$\cos\left(\frac{y}{x}\right) + \log_e(\sqrt{x}) = K$, where K is an arbitrary constant

Q.5	Let $f: \mathbb{R}^3 \to \mathbb{R}$ be a continuous function. Then $$\int_0^2 \int_0^x \int_0^y f(x,y,z)\,dz\,dy\,dx \;=\; \_\_\_\_\_.$$
(A)	$\int_0^2 \int_0^y \int_y^2 f(x,y,z)\,dx\,dz\,dy$
(B)	$\int_0^2 \int_0^x \int_z^2 f(x,y,z)\,dy\,dz\,dx$
(C)	$\int_0^2 \int_y^2 \int_0^x f(x,y,z)\,dz\,dx\,dy$
(D)	$\int_0^2 \int_z^2 \int_0^x f(x,y,z)\,dy\,dx\,dz$

Q.6	Let $f:(1,2) \to \mathbb{R}$ be a continuous function satisfying $$\lim_{x\to 1+} f(x) = 3 \text{ and } \lim_{x\to 2-} f(x) = 3.$$ Then which ONE of the following is necessarily TRUE?
(A)	f is bounded and f has a maximum or a minimum but not both.
(B)	f is bounded and f has a maximum or a minimum or both.
(C)	f is bounded and f has neither a maximum nor a minimum.
(D)	f is unbounded and f has either no maximum or no minimum.
Q.7	Let $f:\mathbb{R} \to \mathbb{R}$ be a twice differentiable function such that $f(n) = 1$ for all $n \in \mathbb{Z}$. Which ONE of the following statements is necessarily TRUE?
(A)	$f'(n) = 0$ for all $n \in \mathbb{Z}$.
(B)	$f'(x) = 0$ for infinitely many $x \in \mathbb{R} \setminus \mathbb{Z}$.
(C)	$f''(n) = 0$ for all $n \in \mathbb{Z}$.
(D)	$f''(x) = 0$ for infinitely many $x \in \mathbb{R} \setminus \mathbb{Z}$.

Q.8	Let A be an invertible 5×5 real matrix. Let B be the matrix obtained by interchanging the second and third columns of A and then adding 3 times the third column to the fifth column. Which ONE of the following statements is TRUE?
(A)	B^{-1} is obtained by interchanging the second and third rows of A^{-1}, and then adding 3 times the third row to the fifth row.
(B)	B^{-1} is obtained by interchanging the second and third rows of A^{-1}, and then adding -3 times the fifth row to the third row.
(C)	B^{-1} is obtained by adding -3 times the third row to the fifth row, and then interchanging the second and third rows of A^{-1}.
(D)	B^{-1} is obtained by adding 3 times the fifth row to the third row, and then interchanging the second and third rows of A^{-1}.
Q.9	Which ONE of the following statements is FALSE?
(A)	S_3 is isomorphic to a subgroup of S_4.
(B)	$\mathbb{Z}_3$ is isomorphic to a subgroup of S_4.
(C)	S_3 is isomorphic to a quotient group of S_4.
(D)	$\mathbb{Z}_6$ is isomorphic to a quotient group of S_4.

Q.10	Let T be the triangular region in the plane with vertices at $(0,0)$, $(1,0)$ and $(1,1)$. Let f be a function defined from T to $\mathbb{R}$ given in polar coordinates. Then the double integral of f over T is equal to ______.
(A)	$\int_0^{\sqrt{2}}\left(\int_0^{\pi/4} f\ d\theta\right) r\,dr + \int_1^{\sqrt{2}}\left(\int_0^{\cos^{-1}(1/r)} f\ d\theta\right) rdr$
(B)	$\int_0^{\sqrt{2}}\left(\int_0^{\pi/4} f\ d\theta\right) rdr - \int_1^{\sqrt{2}}\left(\int_0^{\cos^{-1}(1/r)} f\ d\theta\right) rdr$
(C)	$\int_0^{\sqrt{2}}\left(\int_0^{\pi/4} f\ d\theta\right) r\,dr + \int_1^{\sqrt{2}}\left(\int_0^{\sin^{-1}(1/r)} f\ d\theta\right) rdr$
(D)	$\int_0^{\sqrt{2}}\left(\int_0^{\pi/4} f\ d\theta\right) r\,dr - \int_1^{\sqrt{2}}\left(\int_0^{\sin^{-1}(1/r)} f\ d\theta\right) rdr$

Section A: Q.11 – Q.30 Carry TWO marks each.	
Q.11	Given a sequence of real numbers (a_n), define $b_n = \begin{cases} a_n, & \text{if } a_n \geq 0 \\ 0, & \text{if } a_n < 0 \end{cases}$ and $c_n = \begin{cases} a_n, & \text{if } a_n < 0 \\ 0, & \text{if } a_n \geq 0 \end{cases}$ for each positive integer n. Which ONE of the following statements is TRUE?
(A)	If $\sum_{n=1}^{\infty} a_n$ does not converge, then $\sum_{n=1}^{\infty} b_n$ does not converge.
(B)	If $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} \lvert a_n \rvert$ does not converge, then $\sum_{n=1}^{\infty} b_n$ converges.
(C)	If $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} \lvert a_n \rvert$ does not converge, then $\sum_{n=1}^{\infty} b_n$ does not converge.
(D)	If $\sum_{n=1}^{\infty} a_n$ converges, then $\sum_{n=1}^{\infty} b_n$ converges or $\sum_{n=1}^{\infty} c_n$ converges.

Q.12	For which one of the following pairs of values of a and b, does the series $\sum_{k=1}^{\infty} \frac{(3x)^k}{2^{\sqrt{k}}(1-x)^k}$ converge for all $x \in (a,b)$?
(A)	$a = \frac{-1}{4}$ and $b = \frac{1}{2}$
(B)	$a = -1$ and $b = \frac{1}{5}$
(C)	$a = \frac{-1}{2}$ and $b = \frac{1}{4}$
(D)	$a = \frac{-1}{4}$ and $b = \frac{1}{3}$
Q.13	Let $f: \mathbb{R} \to \mathbb{R}$ be a function that has derivatives of all orders. Which ONE of the following statements is FALSE?
(A)	If f is a polynomial in x of degree at most 5, then $f^{(6)}(x) = 0$ for all $x \in \mathbb{R}$.
(B)	If $f^{(6)}(x) = 0$ for all $x \in \mathbb{R}$, then f is a polynomial in x of degree at most 5.
(C)	If $f^{(k)}(a) = 0$ for all $1 \leq k \leq 5$ and $f^{(6)}(a) > 0$ for some $a \in \mathbb{R}$, then $f(x)$ has a local minimum at a.
(D)	If $f^{(k)}(a) = 0$ for all $1 \leq k \leq 6$ and $f^{(7)}(a) < 0$ for some $a \in \mathbb{R}$, then $f(x)$ has a local maximum at a.

Q.14	Let A be a 5×3 real matrix of rank 2 and b be a non-zero 5×1 real column vector. Suppose $\begin{pmatrix}1\\2\\3\end{pmatrix}$ and $\begin{pmatrix}4\\5\\6\end{pmatrix}$ are two solutions of the system of linear equations $Ax = b$. Which ONE of the following is also a solution of $Ax = b$?
(A)	$\begin{pmatrix}3\\3\\3\end{pmatrix}$
(B)	$\begin{pmatrix}5\\7\\9\end{pmatrix}$
(C)	$\begin{pmatrix}5\\6\\9\end{pmatrix}$
(D)	$\begin{pmatrix}7\\8\\9\end{pmatrix}$

Q.15	Let $f : \mathbb{R}^2 \to \mathbb{R}$ be the function given by $f(x,y) = (x^2-1)^2 + (y^2-1)^2$ for all $(x,y) \in \mathbb{R}^2$. Which ONE of the following statements is TRUE?
(A)	f has local maxima at exactly two points.
(B)	f has local minima at exactly two points.
(C)	f has local minima at exactly three points.
(D)	f has exactly four saddle points.
Q.16	The orthogonal trajectories of the family of curves $y = -3x - 3 + me^x$ for a real parameter m are given by _______.
(A)	$x = \frac{y}{3} - \frac{1}{9} + k\, e^{-3y}$, k is a real parameter
(B)	$x = \frac{y}{3} - \frac{1}{9} + k\, e^{3y}$, k is a real parameter
(C)	$x = \frac{-y}{3} + \frac{1}{9} + k\, e^{-3y}$, k is a real parameter
(D)	$x = \frac{-y}{3} + \frac{1}{9} + k\, e^{3y}$, k is a real parameter

Q.17	Let $f : \mathbb{R}^2 \to \mathbb{R}$ be the function defined by $$f(x,y) = \begin{cases} \dfrac{5x^2y - 4y^3}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$ Which ONE of the following statements is FALSE?
(A)	f is continuous at $(0,0)$
(B)	$\dfrac{\partial f}{\partial x}$ at $(0,1) = 0$
(C)	$\dfrac{\partial f}{\partial y}$ at $(1,0) = 5$
(D)	$\dfrac{\partial f}{\partial y}$ is continuous at $(0,0)$

Q.18	Let $\phi : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$ be the solution of the differential equation $$(\cos x)\frac{dy}{dx} - y = 2y^2(\sin x - 1)\cos x$$ satisfying $\phi(0) = \frac{1}{2}$. Then $\phi\left(\frac{\pi}{4}\right) =$______.
(A)	2
(B)	$\frac{3+2\sqrt{2}}{2}$
(C)	$\frac{1}{2}$
(D)	$\frac{1}{\sqrt{2}}$

Q.19	The value of the following expression $$\lim_{x\to\infty}\int_0^x e^{-t}\lvert\sin t\rvert\,dt$$ is _____.
(A)	$\dfrac{e^{\pi}+1}{2(e^{\pi}-1)}$
(B)	$\dfrac{e^{\pi}-1}{2(e^{\pi}+1)}$
(C)	$\dfrac{2(e^{\pi}+1)}{e^{\pi}-1}$
(D)	$\dfrac{2(e^{\pi}-1)}{e^{\pi}+1}$
Q.20	The semi-circle $y=\sqrt{6x-5-x^2}$ is rotated clockwise by the angle $\frac{\pi}{2}$ in the plane about the origin. What is the area of the planar region traced out by the semi-circle?
(A)	6π
(B)	4π
(C)	2π
(D)	π

Q.21	Let $f: [1,9] \to \mathbb{R}$ be a non-constant continuous function such that $f(1) = f(9)$. Which ONE of the following statements is necessarily TRUE?
(A)	There exists $c \in [4,8]$ such that $f(c) = f(c+1)$.
(B)	There exists $c \in [3,7]$ such that $f(c) = f(c+2)$.
(C)	There exists $c \in [2,6]$ such that $f(c) = f(c+3)$.
(D)	There exists $c \in [1,5]$ such that $f(c) = f(c+4)$.
Q.22	Let S be the solid of intersection of two solid spheres S_1, S_2 given by $S_1: x^2 + y^2 + (z-1)^2 \leq 4 \quad \text{and} \quad S_2: x^2 + y^2 + (z+1)^2 \leq 4$. Which ONE of the following iterated integrals expresses the volume of S?
(A)	$8\int_0^{\sqrt{3}} \int_0^{\sqrt{3-x^2}} \left(\sqrt{4-x^2-y^2} - 1\right) dy\, dx$
(B)	$8\int_0^{2} \int_0^{\sqrt{4-x^2}} \left(\sqrt{4-x^2-y^2} - 1\right) dy\, dx$
(C)	$2\int_0^{2} \int_0^{\sqrt{4-x^2}} \left(\sqrt{4-x^2-y^2} - 1\right) dy\, dx$
(D)	$2\int_0^{\sqrt{3}} \int_0^{\sqrt{3-x^2}} \left(\sqrt{4-x^2-y^2} - 1\right) dy\, dx$

Q.23	Consider the differential equation $$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = \sin(e^x).$$ Which ONE of the following is a particular solution of this differential equation?
(A)	$-e^{-x}\sin(e^x)$
(B)	$-e^{-2x}\sin(e^x)$
(C)	$-e^{-x}(\sin(e^x) + \cos(e^x))$
(D)	$-e^{-2x}(\sin(e^x) + \cos(e^x))$
Q.24	Let e_1, e_2, e_3, e_4, e_5 denote the standard basis vectors of the real vector space $\mathbb{R}^5$. Let V be the subspace of $\mathbb{R}^5$ spanned by the vectors $e_1 + e_2, e_2 + e_3$, $e_3 + e_4 + e_5$. Consider the real vector space $M_{2,5}(\mathbb{R})$. Let $$S = \{P \in M_{2,5}(\mathbb{R}) \mid \text{nullspace}(P) = V\}.$$ Which ONE of the following statements is TRUE?
(A)	S is the empty set.
(B)	$S = \{0\}$.
(C)	S is non-empty and S is not a subspace of $M_{2,5}(\mathbb{R})$.
(D)	S is a nonzero subspace of $M_{2,5}(\mathbb{R})$.

Q.25	Let P be a nonzero complex $n \times n$ matrix such that $v^*Pv \geq 0$ for all column vectors v in $\mathbb{C}^n$. Which ONE of the following statements is necessarily TRUE?
(A)	All eigenvalues of P are negative real numbers.
(B)	If v_1, v_2 are column vectors in $\mathbb{C}^n$ such that $Pv_1 = v_2$ and $Pv_2 = v_1$, then $v_1 = v_2$.
(C)	$v^*Pv \neq 0$ for all nonzero column vectors v in $\mathbb{C}^n$.
(D)	All eigenvalues of P^* are negative real numbers.
Q.26	Let $f: G \longrightarrow \mathbb{Z}$ be a surjective group homomorphism from an abelian group G to the additive group of integers $\mathbb{Z}$. Let K denote the kernel of f. Which ONE of the following statements is FALSE?
(A)	There exists a group homomorphism $\phi: \mathrm{Z} \longrightarrow G$ such that $\mathrm{f} \circ \phi$ is the identity homomorphism.
(B)	K is isomorphic to a quotient group of G.
(C)	For all non-zero homomorphisms $\alpha: \mathrm{Z} \longrightarrow G$ and $\beta: \mathrm{G} \longrightarrow K$, the composition $\beta \circ \alpha$ is non-zero.
(D)	G has a subgroup isomorphic to $\mathbb{Z}$.

Q.27	Which ONE of the following statements is FALSE?
(A)	There is a surjective group homomorphism from the additive group of rational numbers to the multiplicative group of all complex roots of unity.
(B)	The multiplicative group of complex numbers of modulus one is isomorphic to a quotient group of the additive group of real numbers.
(C)	Any group homomorphism from the multiplicative group of nonzero complex numbers into the group of all invertible 2×2 matrices with real entries has nontrivial kernel.
(D)	There exists a group homomorphism from the symmetric group on n symbols into the multiplicative group of all invertible $n \times n$ matrices with real entries, which has a trivial kernel.

Q.28	For any two points $P, Q \in \mathbb{R}^3$, let $vect(P, Q)$ denote the vector from the point P to the point Q and let $d(P, Q)$ denote the length of $vect(P, Q)$. Let $F: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be a linear transformation such that $$d\big(F(P), F(Q)\big) = d(P, Q) \quad \text{for all } P, Q \in \mathbb{R}^3.$$ Which ONE of the following statements is FALSE ?
(A)	F is an injective map.
(B)	F is a surjective map.
(C)	For any four points P, Q, R, S in $\mathbb{R}^3$, we have $vect\big(F(P), F(Q)\big) \cdot vect\big(F(R), F(S)\big) = vect(P, Q) \cdot vect(R, S)$, where $\cdot$ denotes the usual dot product in $\mathbb{R}^3$.
(D)	For any four points P, Q, R, S in $\mathbb{R}^3$, we have $vect\big(F(P), F(Q)\big) \times vect\big(F(R), F(S)\big) = vect(P, Q) \times vect(R, S)$, where $\times$ denotes the usual cross product in $\mathbb{R}^3$.

Q.29	For each positive integer n, let $$x_n = 1 + \frac{1}{2} + \cdots + \frac{1}{n} - \log_e n$$ and $$y_n = \int_1^n \frac{\cos t}{t^2}\, dt\ .$$ Which ONE of the following statements about the sequences (x_n) and (y_n) is TRUE?
(A)	(x_n) and (y_n) are convergent.
(B)	(x_n) is convergent and (y_n) is not convergent.
(C)	(x_n) is not convergent and (y_n) is convergent.
(D)	(x_n) is not convergent and (y_n) is not convergent.
Q.30	Let G be the power set of $\{1,2,3,4\}$. Let the binary operation Δ on G be $$A\,\Delta\, B = (A \setminus B) \cup (B \setminus A).$$ Which ONE of the following statements is TRUE?
(A)	$\{1\}$ is an identity element of (G, Δ).
(B)	(G, Δ) is an abelian group but not cyclic.
(C)	(G, Δ) is a group and has an element of order 4.
(D)	(G, Δ) is a group and has an element of order 8.

Section B: Q.31 – Q.40 Carry TWO marks each.	
Q.31	Which of the following functions $f\colon \mathbb{R} \to \mathbb{R}$ has/have a local minimum at $x = 0$?
(A)	$f(x) = \sin\lvert x\rvert$
(B)	$f(x) = \sin x + \frac{x^3}{6}$
(C)	$f(x) = x^4 + x^2 + 3$
(D)	$f(x) = \min\{x - [x], 1 - x + [x]\}$
Q.32	Which of the following statements is/are TRUE?
(A)	There exists a monotone sequence that does not converge but has a convergent subsequence.
(B)	There exists a sequence that has a bounded subsequence but does not have any convergent subsequence.
(C)	There exists a sequence (x_n) such that given any positive integer m, (x_n) has a subsequence converging to m.
(D)	There exists a sequence (x_n) such that $(\lvert x_{n+1} - x_n\rvert)$ converges to 0 but (x_n) does not converge.

Q.33	Which of the following functions is/are differentiable at $x = 1$?
(A)	$f(x) = \|x - 1\|^3$
(B)	$f(x) = \|x^2 - 1\|$
(C)	$f(x) = \begin{cases} x^2 e^{-x^2}, & \|x\| \leq 1 \\ e^{-1}, & \|x\| > 1 \end{cases}$
(D)	$f(x) = [x]$
Q.34	Let $f: \mathbb{R} \to \mathbb{R}$ be the function given by $$f(x) = \begin{cases} \cos x, & x \in \mathbb{Q} \\ 0, & x \notin \mathbb{Q}. \end{cases}$$ Which of the following statements is/are TRUE?
(A)	$f(x)$ is continuous at 0.
(B)	$f(x)$ is continuous at $\frac{\pi}{2}$.
(C)	$f(x)$ is Riemann integrable on [0, 1] and $\int_0^1 f(x)\,dx = \sin 1$.
(D)	$f(x)$ is Riemann integrable on [0, 1] and $\int_0^1 f(x)\,dx = 0$.

Q.35	Consider the differential equation $$(2y\cos x - xy\ \sin x)\,dx + 2x\cos x\,dy = 0 \quad \text{for } x \in \left(0, \frac{\pi}{4}\right).$$ Which of the following is/are integrating factor(s) of the differential equation?
(A)	$\frac{1}{xy}$
(B)	xy
(C)	$\sec x$
(D)	$\sqrt{\sec x}$
Q.36	Let $f(x,y) = \begin{cases} \frac{x^2 y}{1+x^2}\sin(1/x)\,, & x \neq 0 \\ 0 \quad\quad , & x = 0 \end{cases}$ Which of the following statements is/are TRUE?
(A)	$\lim\limits_{y\to 0}\lim\limits_{x\to 0} f(x,y)$ exists.
(B)	$\frac{\partial f}{\partial x}$ is continuous at the point $(0,1)$.
(C)	$\frac{\partial f}{\partial y}$ is continuous at the point $(0,1)$.
(D)	$\lim\limits_{(x,y)\to(0,0)} \frac{f(x,y)}{\sqrt{x^2+y^2}}$ does not exist.

Q.37	Let $P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \in M_5(\mathbb{C})$. Which of the following statements is/are TRUE?
(A)	$\text{nullity}(P - I) \geq 2$, where I is the 5×5 identity matrix.
(B)	P has 4 distinct eigenvalues in $\mathbb{C}$.
(C)	P has 3 distinct eigenvalues in $\mathbb{R}$.
(D)	If λ is an eigenvalue of P, then there exists a positive integer n such that $\lambda^n = 1$.

Q.38	Let $\binom{n}{r}$ denote the number of ways of choosing r distinct objects out of n distinct objects. Which of the following statements is/are TRUE?
(A)	$\sum_{k=0}^{6}(-1)^k\binom{13}{2k} = -64.$
(B)	$\sum_{k=0}^{6}(-1)^k\binom{13}{2k+1} = 128.$
(C)	$\sum_{k=0}^{6}(-1)^k\binom{13}{2k} = 64.$
(D)	$\sum_{k=0}^{6}(-1)^k\binom{13}{2k+1} = -128.$
Q.39	Consider matrices P of order 4×6 and Q of order 6×4 with real entries such that $PQ = 0$ and $QP = 0$. Which of the following statements is/are TRUE?
(A)	$\text{rangespace}(P) \subseteq \text{nullspace}(Q)$ and $\text{rangespace}(Q) \subseteq \text{nullspace}(P)$.
(B)	$\text{rank}(P) + \text{rank}(Q) \leq 4.$
(C)	If $\text{rangespace}(P) = \text{nullspace}(Q)$, then $\text{rank}(P) + \text{rank}(Q) = 4.$
(D)	$\text{rangespace}(Q) = \text{nullspace}(P).$

Q.40	Let V be the subset of $\mathbb{R}$ defined by $$V = \left\{\frac{a+b\sqrt{2}}{c+d\sqrt{2}} : a,b,c,d \in \mathbb{Q}, c^2+d^2 \neq 0\right\}.$$ Which of the following statements is/are FALSE?
(A)	V is closed under the usual addition in $\mathbb{R}$.
(B)	V is a subspace of the real vector space $\mathbb{R}$.
(C)	V is a two dimensional subspace of the vector space $\mathbb{R}$ over $\mathbb{Q}$.
(D)	V is a four dimensional subspace of the vector space $\mathbb{R}$ over $\mathbb{Q}$.
Section C: Q.41 – Q.50 Carry ONE mark each.	
Q.41	The radius of convergence of the series $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} (\log_e n)^{-1} x^n$ is ______ (*rounded off to one decimal place*).
Q.42	$\int_0^1 \left(\sum_{k=1}^{\infty} \frac{(\log_e 2)^k x^{k-1}}{(k-1)!}\right) dx =$ ______(*rounded off to one decimal place*).

Q.43	$\lim_{n\to\infty}\left(\frac{n^2+1}{\sqrt{n^6+1}}+\cdots+\frac{n^2+n}{\sqrt{n^6+n}}\right)=$ ______(*rounded off to one decimal place*).
Q.44	$\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{\sin x}+e^{\frac{1-\cos x}{x}}\right)=$ ______(*rounded off to one decimal place*).
Q.45	Let α and β be real numbers such that the differential equation $(y^3+\alpha x y^4-5x+\cos 2y)dx+(3xy^2+20x^2y^3+\beta x\sin 2y)dy=0$ is exact. Then $\alpha+\beta=$ ______(*rounded off to one decimal place*).
Q.46	Let $z=\cos(4x+5y)$, where $x=\frac{\pi}{2}+2\theta,\ y=-\left(\frac{\pi}{4}+\theta\right)$. Then $\left.\frac{dz}{d\theta}\right\|_{\theta=\frac{\pi}{4}}=$ ______(*rounded off to one decimal place*).

Q.47	Let C_n denote a cyclic group having n elements. If there is a surjective group homomorphism from C_n to C_{30}, then the total number of such distinct surjective homomorphisms is ______.
Q.48	$\int_0^3 (\lvert x-1 \rvert - x[x])\, dx$ = ______(*rounded off to one decimal place*).
Q.49	Let $A \in M_3(\mathbb{C})$. Suppose the column vector $v = \begin{pmatrix} \sqrt{5}i \\ 2i \\ x \end{pmatrix}$ in $\mathbb{C}^3$ belongs to the intersection of nullspace(A) and rangespace(A^T). Then $\lvert x \rvert$ = ______(*rounded off to one decimal place*).
Q.50	Let P be a 5×5 real matrix with $\det(P) = 2$. Let Q be the matrix of cofactors of P. Then $\det(Q)$ = ________ (*rounded off to one decimal place*).

Section C: Q.51 – Q.60 Carry TWO marks each.

Q.51 Let α, β and γ be fixed real numbers such that

$y(x) = C_1 e^{-x} + C_2 e^{2x} + \alpha x e^{-x}$, where C_1 and C_2 are arbitrary real constants,

is the general solution of the differential equation

$$\frac{d^2y}{dx^2} + \beta \frac{dy}{dx} + \gamma y = -e^{-x}.$$

Then $\alpha(\beta + \gamma)$= ______(*rounded off to one decimal place*).

Q.52 Let the function $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} 4x^2 \tan^{-1}\left(\frac{y}{2x}\right) + y^3 \tan^{-1}\left(\frac{x}{4y^2}\right), & xy \neq 0 \\ 0, & \text{otherwise} \end{cases}$$

Then $\left(\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right)\right)$ at $(0,0)$ is ______ (*rounded off to two decimal places*).

Q.53 The double integral of $f(x,y) = x$ over the triangular region with vertices at $\left(-\frac{1}{2}, \frac{1}{2}\right)$, $(1,2)$ and $(1,-1)$ is ______(*rounded off to one decimal place*).

Q.54	Let $$f(x) = \int_0^{e^{\sin x}} \int_0^{\log_e y} e^{-\left(\frac{1}{\sqrt{2}}\right)} (1 - t^2)\, dt\, dy.$$ Then $f'(\pi/4) =$ ______(*rounded off to one decimal place*).
Q.55	The volume of the tetrahedron bounded by the planes $x = 1, y = 2, z = 3$ and $12x + 8y + 6z = 70$ is ______(*rounded off to one decimal place*).
Q.56	Let $M \in M_3(\mathbb{R})$. If $$\mathrm{M}\begin{pmatrix} \sqrt{15} \\ \sqrt{15} \\ 0 \end{pmatrix} = \begin{pmatrix} \sqrt{3} \\ \sqrt{2} \\ -5 \end{pmatrix}, \quad M\begin{pmatrix} 0 \\ 0 \\ \sqrt{30} \end{pmatrix} = \begin{pmatrix} \sqrt{15} \\ \sqrt{10} \\ \sqrt{5} \end{pmatrix}, \quad M\begin{pmatrix} -\sqrt{15} \\ \sqrt{15} \\ 0 \end{pmatrix} = \begin{pmatrix} \sqrt{12} \\ -\sqrt{18} \\ 0 \end{pmatrix}$$ then $\det(M)$ = ______(*rounded off to one decimal place*).

Q.57	Let $a, b \in \mathbb{R}$. If the system of linear equations $x + 2y + 2z = 1$ $2x + 3y + z = 2$ $ax + 5y + bz = b$ has infinitely many solutions, then $a + b =$ ______ (*rounded off to one decimal place*).
Q.58	A fruit shop has 4 different types of bananas. The number of ways in which 12 bananas can be bought with at least one banana from each type, is ______.
Q.59	Let A be a 3×3 real matrix such that given any column vector $x \in \mathbb{R}^3$, the column vector Ax is the reflection of x about the plane $\{(a, b, -a - b): a, b \in \mathbb{R}\}$. Then the sum of the diagonal elements of A is ______(*rounded off to one decimal place*).

Q.60	Let $\binom{n}{r}$ denote the number of ways of choosing r distinct objects out of n distinct objects. Then, $$\binom{5}{0}+\binom{6}{1}+\binom{7}{2}+\binom{8}{3}+\binom{9}{4}+\binom{10}{5}+\binom{11}{6}=\_\_\_\_\_.$$