JAM 2015: General Instructions during Examination

1. Total duration of the JAM 2015 examination is **180** minutes.
2. The clock will be set at the server. The countdown timer at the top right corner of screen will display the remaining time available for you to complete the examination. When the timer reaches zero, the examination will end by itself. You need not terminate the examination or submit your paper.
3. Any useful data required for your paper can be viewed by clicking on the **Useful Data** button that appears on the screen.
4. Use the scribble pad provided to you for any rough work. Submit the scribble pad at the end of the examination.
5. You are allowed to use only your own **non-programmable calculator**.
6. The Question Palette displayed on the right side of screen will show the status of each question using one of the following symbols:

You have not visited the question yet.

You have not answered the question.

You have answered the question.

You have NOT answered the question, but have marked the question for review.

You have answered the question, but marked it for review.

7. The **Marked for Review** status for a question simply indicates that you would like to look at that question again. *If a question is 'answered, but marked for review', then the answer will be considered for evaluation unless the status is modified by the candidate.*

<u>Navigating to a Question</u> :

8. To answer a question, do the following:
   a. Click on the question number in the Question Palette to go to that question directly.
   b. Select the answer for a multiple choice type question and for the multiple select type question. Use the virtual numeric keypad to enter the answer for a numerical type question.
   c. Click on **Save & Next** to save your answer for the current question and then go to the next question.
   d. Click on **Mark for Review & Next** to save and to mark for review your answer for the current question, and then go to the next question.

   Caution: Note that your answer for the current question will not be saved, if you navigate to another question directly by clicking on a question number without saving the answer to the previous question.
9. You can view all the questions by clicking on the **Question Paper** button. This feature is provided, so that if you want you can just see the entire question paper at a glance.

<u>Answering a Question</u> :

10. <u>Procedure for answering a multiple choice question (MCQ)</u>:
    a. Choose the answer by selecting only one out of the 4 choices (A,B,C,D) given below the question and click on the bubble placed before the selected choice.

b. To deselect your chosen answer, click on the bubble of the selected choice again or click on the **Clear Response** button.

c. To change your chosen answer, click on the bubble of another choice.

d. To save your answer, you MUST click on the **Save & Next** button.

11. Procedure for answering a multiple select question (MSQ):

a. Choose the answer by selecting one or more than one out of the 4 choices (A,B,C,D) given below the question and click on the checkbox(es) placed before each of the selected choice (s).

b. To deselect one or more of your selected choice(s), click on the checkbox(es) of the choice(s) again. To deselect all the selected choices, click on the **Clear Response** button.

c. To change a particular selected choice, deselect this choice that you want to change and click on the checkbox of another choice.

d. To save your answer, you MUST click on the **Save & Next** button.

12. Procedure for answering a numerical answer type (NAT) question:

a. To enter a number as your answer, use the virtual numerical keypad.

b. A fraction (e.g. -0.3 or -.3) can be entered as an answer with or without '0' before the decimal point. As many as four decimal points, e.g. 12.5435 or 0.003 or -932.6711 or 12.82 can be entered.

c. To clear your answer, click on the **Clear Response** button.

d. To save your answer, you MUST click on the **Save & Next** button.

13. To mark a question for review, click on the **Mark for Review & Next** button. *If an answer is selected (for MCQ and MSQ types) or entered (for NAT) for a question that is* ***Marked for Review****, that answer will be considered in the evaluation unless the status is modified by the candidate.*

14. To change your answer to a question that has already been answered, first select that question and then follow the procedure for answering that type of question as described above.

15. Note that ONLY those questions for which answers are ***saved*** or ***marked for review after answering*** will be considered for evaluation.

Choosing a Section :

16. Sections in this question paper are displayed on the top bar of the screen. All sections are compulsory.

17. Questions in a section can be viewed by clicking on the name of that section. The section you are currently viewing will be highlighted.

18. To select another section, simply click the name of the section on the top bar. You can shuffle between different sections any number of times.

19. When you select a section, you will only be able to see questions in this Section, and you can answer questions in the Section.

20. After clicking the **Save & Next** button for the last question in a section, you will automatically be taken to the first question of the next section in sequence.

21. You can move the mouse cursor over the name of a section to view the answering status for that section.

JAM 2015 Examination

MS: Mathematical Statistics

Duration: **180 minutes** *Maximum Marks*: **100**

Read the following instructions carefully.

1. To login, enter your Registration Number and Password provided to you. Kindly go through the various coloured symbols used in the test and understand their meaning before you start the examination.
2. Once you login and after the start of the examination, you can view all the questions in the question paper, by clicking on the **Question Paper** button in the screen.
3. This test paper has a total of 60 questions carrying 100 marks. The entire question paper is divided into three **sections, A, B** and **C**. All sections are compulsory. Questions in each section are of different types.
4. **Section – A** contains **Multiple Choice Questions (MCQ)**. Each MCQ type question has four choices out of which only one choice is the correct answer. This section has 30 Questions and carry a total of 50 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.30 carry 2 marks each.
5. **Section – B** contains **Multiple Select Questions (MSQ)**. Each MSQ type question is similar to MCQ but with a difference that there may be one or more than one choice(s) that are correct out of the four given choices. The candidate gets full credit if he/she selects all the correct choices only and no wrong choices. This section has 10 Questions and carry 2 marks each with a total of 20 marks.
6. **Section – C** contains **Numerical Answer Type (NAT)** questions. For these NAT type questions, the answer is a real number which needs to be entered using the virtual numerical keypad on the monitor. No choices will be shown for these type of questions. This section has 20 Questions and carry a total of 30 marks. Q.1 – Q.10 carry 1 mark each and Questions Q.11 – Q.20 carry 2 marks each.
7. Depending upon the JAM test paper, there may be useful common data that may be required for answering the questions. If the paper has such useful data, the same can be viewed by clicking on the **Useful Data** button that appears at the top, right hand side of the screen.
8. The computer allotted to you at the examination centre runs specialized software that permits only one choice to be selected as answer for multiple choice questions using a mouse, one or more than one choices to be selected as answer for multiple select questions using a mouse and to enter a suitable number for the numerical answer type questions using the virtual numeric keypad and mouse.
9. Your answers shall be updated and saved on a server periodically and also at the end of the examination. The examination will **stop automatically** at the end of **180 minutes**.
10. Multiple choice questions (Section-A) will have four choices against A, B, C, D, out of which only **ONE** choice is the correct answer. The candidate has to choose the correct answer by clicking on the bubble (○) placed before the choice.
11. Multiple select questions (Section-B) will also have four choices against A, B, C, D, out of which **ONE OR MORE THAN ONE** choice(s) is /are the correct answer. The candidate has to choose the correct answer by clicking on the checkbox (□) placed before the choices for each of the selected choice(s).
12. For numerical answer type questions (Section-C), each question will have a numerical answer and there will not be any choices**. For these questions, the answer should be entered** by using the mouse and the virtual numerical keypad that appears on the monitor.
13. In all questions, questions not attempted will result in zero mark. In **Section – A** (MCQ), wrong answer will result in **NEGATIVE** marks. For all 1 mark questions, 1/3 marks will be deducted for each wrong answer. For all 2 marks questions, 2/3 marks will be deducted for each wrong answer. In **Section – B** (MSQ), there is **NO NEGATIVE** and **NO PARTIAL** marking provisions. There is **NO NEGATIVE** marking in **Section – C** (NAT) as well.

14. **Non-programmable calculators** are allowed but sharing of calculators is not allowed.

15. Mobile phones, electronic gadgets other than calculators, charts, graph sheets, and mathematical tables are **NOT** allowed in the examination hall.

16. You can use the scribble pad provided to you at the examination centre for all your rough work. The scribble pad has to be returned at the end of the examination.

Declaration by the candidate:

"I have read and understood all the above instructions. I have also read and understood clearly the instructions given on the admit card and shall follow the same. I also understand that in case I am found to violate any of these instructions, my candidature is liable to be cancelled. I also confirm that at the start of the examination all the computer hardware allotted to me are in proper working condition".

Special Instructions / Useful Data	
$\mathbb{R}$	Set of all real numbers
$\mathbb{R}^n$	$\{(x_1, \ldots, x_n) : x_i \in \mathbb{R},\ i = 1, 2, \ldots, n\}$
$E(X)$	Expectation of the random variable X
$P(A)$	Probability of the event A
$\bar{X}_n$	$\frac{1}{n}\sum_{i=1}^{n} X_i$
$U(a,b)$	Continuous uniform distribution on (a,b), $-\infty < a < b < \infty$
$N(\mu,\sigma^2)$	Normal distribution with mean $\mu \in \mathbb{R}$ and variance $\sigma^2 > 0$
$Bin(n, p)$	Binomial distribution with n trials and success probability p
$Poisson(\theta)$	Poisson distribution with parameter θ
$Geom(p)$	Geometric distribution with parameter p, whose probability mass function is given by $P(X = x) = p(1-p)^{x-1}$, $x = 1, 2, \ldots$
$Gamma(\alpha,\beta)$	Gamma distribution with parameters α and β, whose probability density function is given by $f(x) = \begin{cases} \dfrac{x^{\alpha-1}\, e^{-x/\beta}}{\Gamma(\alpha)\, \beta^{\alpha}}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$
$X_n \xrightarrow{P} X$	The sequence of random variables $\{X_n\}$ converges in probability to the random variable X
$X_n \xrightarrow{d} X$	The sequence of random variables $\{X_n\}$ converges in distribution to the random variable X
$\binom{n}{x}$	Binomial coefficient, equal to $\dfrac{n!}{x!\,(n-x)!}$
$\log(x)$	Natural logarithm of x
I	Identity matrix
$\Phi(x)$	Cumulative distribution function of $N(0,1)$
Special values	$\Phi(1.285) = 0.900$, $\Phi(1.645) = 0.950$, $\Phi(1.96) = 0.975$ $e = 2.718$, $\pi = 3.142$ $\log(2) = 0.693$

SECTION – A

MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 Let $X_1, \ldots, X_n$ be a random sample from a population with probability density function

$$f_\theta(x) = \begin{cases} \theta e^{-\theta x}, & x > 0, \\ 0, & \text{otherwise,} \end{cases}$$

where $\theta > 0$ is an unknown parameter.

Then, the uniformly minimum variance unbiased estimator for $\dfrac{1}{\theta}$ is

(A) $\dfrac{1}{\bar{X}_n}$

(B) $\sum_{i=1}^{n} X_i$

(C) $\bar{X}_n$

(D) $\dfrac{1}{\sum_{i=1}^{n} X_i}$

Q.2 Let $X_1, \ldots, X_{100}$ be independent and identically distributed $N(0,1)$ random variables. The correlation between $\sum_{i=1}^{98} X_i$ and $\sum_{i=3}^{100} X_i$ is equal to

(A) 0 (B) $96/98$ (C) $98/100$ (D) 1

Q.3 Consider the problem of testing $H_0 : \theta = 0$ against $H_1 : \theta = 1/2$ based on a single observation X from $U(\theta, \theta + 1)$ population. The power of the test "Reject H_0 if $X > \dfrac{2}{3}$" is

(A) $1/6$ (B) $5/6$ (C) $1/3$ (D) $2/3$

Q.4 The probability mass function of a random variable X is given by

$$P(X=x)=k\binom{n}{x},\ x=0,1,\ldots,n,$$

where k is a constant. The moment generating function $M_X(t)$ is

(A) $\dfrac{\left(1+e^t\right)^n}{2^n}$

(B) $\dfrac{2^n}{\left(1+e^t\right)^n}$

(C) $\dfrac{1}{2^n\left(1+e^t\right)^n}$

(D) $2^n\left(1+e^t\right)^n$

Q.5 Suppose A and B are events with $P(A)=0.5$, $P(B)=0.4$ and $P\left(A\cap B^c\right)=0.2$. Then $P\left(B^c \mid A\cup B\right)$ is equal to

(A) $1/2$ (B) $1/3$ (C) $1/4$ (D) 0

Q.6 Let $X_1,\ldots,X_n$ be a random sample from a $Gamma(\alpha,\beta)$ population, where $\beta>0$ is a known constant. The rejection region of the most powerful test for $H_0:\alpha=1$ against $H_1:\alpha=2$ is of the form

(A) $\prod_{i=1}^{n} X_i > K$ (B) $\sum_{i=1}^{n} X_i > K$ (C) $\prod_{i=1}^{n} X_i < K$ (D) $\sum_{i=1}^{n} X_i < K$

Q.7 Which of the following is NOT a linear transformation?

(A) $T:\mathbb{R}^3\to\mathbb{R}^2$ defined by $T(x,y,z)=(x,z)$

(B) $T:\mathbb{R}^3\to\mathbb{R}^3$ defined by $T(x,y,z)=(x,y-1,z)$

(C) $T:\mathbb{R}^2\to\mathbb{R}^2$ defined by $T(x,y)=(2x,y-x)$

(D) $T:\mathbb{R}^2\to\mathbb{R}^2$ defined by $T(x,y)=(y,x)$

Q.8 If a sequence $\{x_n\}$ is monotone and bounded, then

(A) there exists a subsequence of $\{x_n\}$ that diverges

(B) there may exist a subsequence of $\{x_n\}$ that is not monotone

(C) all subsequences of $\{x_n\}$ converge to the same limit

(D) there exist at least two subsequences of $\{x_n\}$ which converge to distinct limits

Q.9 Let $f:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=x(x-1)(x-2)$. Then

(A) f is one-one and onto
(B) f is neither one-one nor onto
(C) f is one-one but not onto
(D) f is not one-one but onto

Q.10 Which of the following statements is true for all real numbers x?

(A) $e^{-x}\le 1-x$
(B) $e^{-x}\ge 1-x$
(C) $e^{x}\ge 1-x$
(D) $e^{x}\le 1-x$

Q. 11 – Q. 30 carry two marks each.

Q.11 Let $X_1,\ldots,X_n$ be a random sample from a $Poisson(\theta)$ population, where $\theta>0$ is unknown. The Cramer-Rao lower bound for the variance of any unbiased estimator of $g(\theta)=\theta e^{-\theta}$ equals

(A) $\dfrac{1}{n}\theta(1-\theta)^2 e^{-2\theta}$
(B) $\theta(1-\theta)^2 e^{-2\theta}$
(C) $\theta(1-\theta)e^{-\theta}$
(D) $\dfrac{1}{n}\theta(1-\theta)e^{-\theta}$

Q.12 Let X and Y be two independent random variables such that $X\sim U(0,2)$ and $Y\sim U(1,3)$. Then $P(X<Y)$ equals

(A) $1/2$
(B) $3/4$
(C) $7/8$
(D) 1

Q.13 There are two boxes, each containing two components. Each component is defective with probability 1/4, independent of all other components. The probability that exactly one box contains exactly one defective component equals

(A) $3/8$
(B) $5/8$
(C) $15/32$
(D) $17/32$

Q.14 Consider a normal population with unknown mean μ and variance $\sigma^2=9$. To test $H_0:\mu=0$ against $H_1:\mu\neq 0$, a random sample of size 100 is taken. Based on this sample, the test of the form $\left|\overline{X}_n\right|>K$ rejects the null hypothesis at 5% level of significance. Then, which of the following is a possible 95% confidence interval for μ?

(A) $(-0.488,\ 0.688)$
(B) $(-1.96,\ 1.96)$
(C) $(0.422,\ 1.598)$
(D) $(0.588,\ 1.96)$

Q.15 Let $X_1,\ldots,X_n$ be a random sample from a population with probability density function

$$f_\theta(x)=\begin{cases}(\theta+1)\,x^\theta, & 0<x<1,\\ 0, & \text{otherwise,}\end{cases}$$

where $\theta>0$ is unknown. The maximum likelihood estimator of θ is

(A) $\sum_{i=1}^{n}\log X_i$

(B) $\dfrac{n-\sum_{i=1}^{n}\log X_i}{\sum_{i=1}^{n}\log X_i}$

(C) $\dfrac{n+\sum_{i=1}^{n}\log X_i}{-\sum_{i=1}^{n}\log X_i}$

(D) $\prod_{i=1}^{n}X_i$

Q.16 Let $X_1,\ldots,X_n$ be a random sample from a population with probability density function

$$f_\theta(x)=\begin{cases}\dfrac{4}{\theta}x^3\,e^{-x^4/\theta}, & x>0,\\ 0, & \text{otherwise,}\end{cases}$$

where $\theta>0$ is unknown. Then, a consistent estimator for θ is

(A) $\dfrac{1}{n}\sum_{i=1}^{n}X_i^4$

(B) $\left(\sum_{i=1}^{n}\dfrac{X_i}{n}\right)^4$

(C) $\overline{X}_n$

(D) $\prod_{i=1}^{n}X_i^3$

Q.17 Let the probability density function of a random variable X be given by

$$f(x)=\alpha\,\mathrm{e}^{-x^2-\beta x},\ -\infty<x<\infty.$$

If $E(X)=-\dfrac{1}{2}$, then

(A) $\alpha=\dfrac{1}{\sqrt{\pi}}e^{-1/4}$ and $\beta=1$

(B) $\alpha=\dfrac{1}{\sqrt{\pi}}e^{-1/4}$ and $\beta=-1$

(C) $\alpha=\sqrt{\pi}\,e^{-1/4}$ and $\beta=1$

(D) $\alpha=\sqrt{\pi}\,e^{-1/4}$ and $\beta=-1$

Q.18 Let X be a single observation from a population having an exponential distribution with mean $1/\lambda$. Consider the problem of testing $H_0 : \lambda = 2$ against $H_1 : \lambda = 4$. For the test with rejection region $X \geq 3$, let $\alpha = P(\text{Type I error})$ and $\beta = P(\text{Type II error})$. Then

(A) $\alpha = e^{-6}$ and $\beta = 1 - e^{-12}$
(B) $\alpha = e^{-12}$ and $\beta = 1 - e^{-6}$
(C) $\alpha = 1 - e^{-12}$ and $\beta = e^{-6}$
(D) $\alpha = e^{-6}$ and $\beta = e^{-12}$

Q.19 Let Y be an exponential random variable with mean $1/\theta$, where $\theta > 0$. The conditional distribution of X given Y has Poisson distribution with mean Y. Then, the variance of X is

(A) $\frac{1}{\theta^2}$
(B) $\frac{\theta+1}{\theta}$
(C) $\frac{\theta^2+1}{\theta^2}$
(D) $\frac{\theta+1}{\theta^2}$

Q.20 2000 cashew nuts are mixed thoroughly in flour. The entire mixture is divided into 1000 equal parts and each part is used to make one biscuit. Assume that no cashews are broken in the process. A biscuit is picked at random. The probability that it contains no cashew nuts is

(A) between 0 and 0.1
(B) between 0.1 and 0.2
(C) between 0.2 and 0.3
(D) between 0.3 and 0.4

Q.21 Suppose $X_1, \ldots, X_n$ are independent random variables and $X_k \sim N(0, k\sigma^2)$, $k = 1, \ldots, n$, where σ^2 is unknown. The maximum likelihood estimator for σ^2 is

(A) $\frac{1}{n} \sum_{k=1}^{n} X_k^2$

(B) $\frac{1}{n} \sum_{k=1}^{n} (X_k - \bar{X}_n)^2$

(C) $\frac{1}{n} \sum_{k=1}^{n} \frac{X_k^2}{k}$

(D) $\frac{\sum_{k=1}^{n} X_k^2}{\sum_{k=1}^{n} k}$

Q.22 Let $X_1, \ldots, X_{10}$ be independent and identically distributed $U(-5, 5)$ random variables. Then, the distribution of the random variable $Y = -2 \sum_{i=1}^{10} \log(|X_i|/5)$ is

(A) χ^2_{10}
(B) $10\,\chi^2_{2}$
(C) χ^2_{20}
(D) $\frac{1}{2}\chi^2_{20}$

Q.23 Let $f:\mathbb{R}\to\mathbb{R}$ be a differentiable function so that $f(x)\,f'(x)<0$ for all x. Then, which of the following is necessarily true?

(A) f is an increasing function
(B) f is a decreasing function
(C) $|f|$ is an increasing function
(D) $|f|$ is a decreasing function

Q.24 Let M be the matrix $\begin{pmatrix} 2 & 3 \\ 1 & -1 \end{pmatrix}$. Which of the following matrix equations does M satisfy?

(A) $M^2 + 3M + 5I = 0$
(B) $M^3 - M^2 - 5M = 0$
(C) $M^3 - 3M^2 + M = 0$
(D) $M^2 - M + 5I = 0$

Q.25 If the determinant of an $n\times n$ matrix A is zero, then

(A) $\text{rank}(A) \le n-2$
(B) the trace of A is zero
(C) zero is an eigenvalue of A
(D) $x=0$ is the only solution of $Ax=0$

Q.26 Let $f:(0,\infty)\to\mathbb{R}$ be given by

$$f(x) = \log x - x + 2.$$

Then, the number of roots of f is

(A) 0 (B) 1 (C) 2 (D) 3

Q.27 The number of distinct real values of x for which the matrix

$$\begin{pmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{pmatrix}$$

is singular is

(A) 1 (B) 2 (C) 3 (D) infinite

Q.28 Suppose $f:\mathbb{R}^2 \to \mathbb{R}$ is a continuous function. Define $h:\mathbb{R} \to \mathbb{R}$ by

$$h(x) = \int_0^x \int_0^x f(u,v)\,du\,dv.$$

Then $h'(1)$ is equal to

(A) $2\,f(1,1)$

(B) $f(1,0) + f(0,1)$

(C) $\int_0^1 f(t,t)\,dt$

(D) $\int_0^1 \left(f(1,t) + f(t,1) \right) dt$

Q.29 Let A be a 5×3 real matrix of rank 2. Let $b \in \mathbb{R}^5$ be a non-zero vector that is in the column space of A. Let $S = \left\{ x \in \mathbb{R}^3 : Ax = b \right\}$. Define the translation of a subspace V of $\mathbb{R}^3$ by $x_0 \in \mathbb{R}^3$ as the set $x_0 + V = \left\{ x_0 + v : v \in V \right\}$. Then

(A) S is the empty set

(B) S has only one element

(C) S is a translation of a 1 dimensional subspace

(D) S is a translation of a 2 dimensional subspace

Q.30 Let $f:\mathbb{R} \to \mathbb{R}$ be a differentiable function whose derivative is continuous. Then

$$\lim_{n \to \infty} (n+1) \int_0^1 x^n\, f(x)\,dx$$

(A) is equal to 0

(B) is infinite

(C) is equal to $\int_0^1 f(x)\,dx$

(D) is equal to $f(1)$

SECTION - B

MULTIPLE SELECT QUESTIONS (MSQ)

Q. 1 – Q. 10 carry two marks each.

Q.1 Suppose $\{a_n\}, \{b_n\}$ are sequences such that $a_n > 0, b_n > 0$ for all $n \geq 1$. Given that $\sum a_n$ converges and $\sum b_n$ diverges, which of the following statements is (are) necessarily FALSE?

(A) $\sum (a_n + b_n)$ converges

(B) $\sum \frac{a_n}{b_n}$ converges

(C) $\sum \frac{b_n}{a_n}$ converges

(D) $\sum a_n b_n$ converges

Q.2 Consider the ordinary differential equation

$$x\frac{dy}{dx} + y = x \text{ for } 0 < x < 1.$$

Which of the following is (are) solution(s) to the above?

(A) $y(x) = \frac{x}{2}$

(B) $y(x) = \frac{x}{2} + \frac{2}{x}$

(C) $y(x) = \frac{x}{2} - \frac{2}{x}$

(D) $y(x) = 0$

Q.3 Let $f:[0,1] \to \mathbb{R}$ be a continuous function such that

$$f(0) = -1,\ \ f\left(\frac{1}{2}\right) = 1,\ \ f(1) = -1.$$

Then

(A) f attains the value 0 at least twice in $[0,1]$

(B) f attains the value 0 exactly twice in $[0,1]$

(C) f attains the value 0 exactly once in $[0,1]$

(D) the range of f is $[-1,1]$

Q.4 Let $f:\mathbb{R} \to \mathbb{R}$. Define $g:\mathbb{R} \to \mathbb{R}$ by

$$g(x) = f(x)\left(f(x) + f(-x)\right).$$

Then

(A) g is even for all f

(B) g is odd for all f

(C) g is even if f is even

(D) g is even if f is odd

Q.5 Which of the following matrices can be the variance-covariance matrix of a random vector $X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$?

(A) $\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$

(B) $\begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}$

(C) $\begin{pmatrix} -2 & 1 \\ 1 & 2 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$

Q.6 Let $X_1, \ldots, X_n$ be a random sample from a $N(\theta, 1)$ population, where $-\infty < \theta < \infty$ is unknown. Which of the following statistics is (are) sufficient for θ?

(A) $\sum_{i=1}^{n} X_i$

(B) $\left(X_1, \sum_{i=2}^{n} X_i \right)$

(C) $\left(X_1, X_2 + X_3, \sum_{i=4}^{n} X_i \right)$

(D) $\left(X_1, X_2, X_3, \sum_{i=4}^{n} X_i \right)$

Q.7 Let $X_1, \ldots, X_n$ be a random sample from a $N(\theta, \theta^2)$ distribution, where $\theta > 0$ is unknown. Let

$$T_1 = \sum_{i=1}^{n} X_i \text{ and } T_2 = \sum_{i=1}^{n} X_i^2 .$$

Which of the following statements is (are) correct?

(A) $\dfrac{T_1^2}{n^2}$ is unbiased for θ^2

(B) $\dfrac{T_2}{2n}$ is unbiased for θ^2

(C) $\dfrac{T_1^2}{n^2}$ is consistent for θ^2

(D) $\dfrac{T_2}{2n}$ is consistent for θ^2

Q.8 Suppose X and Y are independent and identically distributed random variables with finite variance σ^2. Which of the following expressions is (are) equal to σ^2?

(A) $E\left(X^2\right) - \left(E(Y)\right)^2$

(B) $E\left(\left(\dfrac{X+Y}{2}\right)^2\right) - \left(E\left(\dfrac{X+Y}{2}\right)\right)^2$

(C) $\dfrac{1}{2} E\left((X-Y)^2\right)$

(D) $\min_{a \in \mathbb{R}} E(X-a)^2$

Q.9 Let $X_1, X_2, \dots$ be a sequence of independent and identically distributed random variables with mean 2 and variance 4. Which of the following statements is (are) true?

(A) $\bar{X}_n \xrightarrow{P} 2$

(B) $\frac{1}{n}\sum_{i=1}^{n} X_i^2 \xrightarrow{P} 4$

(C) $\sqrt{n}\frac{\left(\bar{X}_n - 2\right)}{2} \xrightarrow{d} N(0,1)$

(D) $E\left(\frac{1}{n-1}\sum_{i=1}^{n}\left(X_i - \bar{X}_n\right)^2\right) = 4$ for all $n \geq 2$

Q.10 Let $X_1, \dots, X_n$ (assume $n \geq 2$) be a random sample from an $N\left(\mu, \sigma^2\right)$ population where $-\infty < \mu < \infty$ and $\sigma^2 > 0$ are unknown. Which of the following statements is (are) true?

(A) The maximum likelihood estimator of μ attains the Cramer-Rao lower bound

(B) The uniformly minimum variance unbiased estimator of μ attains the Cramer-Rao lower bound

(C) The maximum likelihood estimator of σ^2 is an unbiased estimator of σ^2

(D) The relative efficiency of the maximum likelihood estimator of σ^2 with respect to the uniformly minimum variance unbiased estimator of σ^2 is strictly less than 1

SECTION – C

NUMERICAL ANSWER TYPE (NAT)

Q. 1 – Q. 10 carry one mark each.

Q.1 Let X and Y be independent exponentially distributed random variables with means $1/4$ and $1/6$ respectively. Let $Z = \min\{X, Y\}$. Then $E(Z) =$ __________.

Q.2 Let X be a $Geom(0.4)$ random variable. Then $P\left(X = 5 \mid X \geq 2\right) =$ __________.

Q.3 X is a single observation from a $Bin(1, p)$ population, where $p \in [1/5, 4/5]$ is unknown. If the observed value of X is 0, then the maximum likelihood estimator of p is __________.

Q.4 X is a random variable with density $f(x) = \frac{1}{4} e^{-|x|/2}, -\infty < x < \infty$. Then $E(|X|) =$ __________.

Q.5 A system comprising of n identical components works if at least one of the components works. Each of the components works with probability 0.8, independent of all other components. The minimum value of n for which the system works with probability at least 0.97 is __________.

Q.6 Let X be a normal random variable with mean 2 and variance 4, and $g(a) = P(a \leq X \leq a+2)$. The value of a that maximizes $g(a)$ is __________.

Q.7 The volume of the solid formed by revolving the curve $y = x$ between $x = 0$ and $x = 1$ about the x-axis is equal to __________.

Q.8 Let $[x]$ be the greatest integer less than or equal to x. Then $\int_{-1}^{2} [x]\, dx =$ __________.

Q.9 The number of real solutions of the equation $x^3 + 3x^2 + 3x + 7 = 0$ is __________.

Q.10 Let $f:\mathbb{R}\to\mathbb{R}$ be a non-constant, three times differentiable function. If $f\left(1+\frac{1}{n}\right)=1$ for all integers n, then $f''(1)=$ ___________.

Q. 11 – Q. 20 carry two marks each.

Q.11 Let $Y \sim U(0,1)$. The conditional probability density function of X given Y is

$$f_{X|Y}(x|y)=\begin{cases}\frac{1}{y}, & \text{if } 0<x<y<1,\\ 0, & \text{otherwise.}\end{cases}$$

Then $E(X)=$ ___________.

Q.12 The probability density function of a random variable X is given by

$$f(x)=\begin{cases}\frac{1}{4}, & \text{if } |x|\le 1,\\ \frac{1}{4x^2}, & \text{otherwise.}\end{cases}$$

Then $P\left(-\frac{1}{2}\le X\le 2\right)=$ ___________.

Q.13 Let $X_1,\ldots,X_n$ be independent and identically distributed random variables with $U(0,1)$ distribution. Then

$\lim_{n\to\infty} P\left(\sum_{i=1}^{n} X_i \le \frac{n}{2}+n^{3/4}\right)=$ ___________.

Q.14 Based on 20 observations $(x_1,y_1),\ldots,(x_{20},y_{20})$, the following values are obtained.

$$\sum_{i=1}^{20} x_i=6,\quad \sum_{i=1}^{20} x_i^2=14,\quad \sum_{i=1}^{20} y_i=9 \quad\text{and}\quad \sum_{i=1}^{20} x_i y_i=20.$$

For $X=1$, the predicted value of Y based on a least squares fit of a linear regression model of Y on X is ___________.

Q.15 The cumulative distribution function of a random variable X is given by

$$F(x)=\begin{cases}0, & \text{if } x<0,\\ \frac{1}{4}+\frac{1}{6}\left(4x-x^2\right), & \text{if } 0\le x<1,\\ 1, & \text{if } x\ge 1.\end{cases}$$

Then $P\left(X=0 \,\middle|\, 0\le X<1\right)=$ __________.

Q.16 The probability density function $f(x)$ of a random variable X is symmetric about 0. Then $\int_{-2}^{2}\int_{-\infty}^{x} f(u)\,du\,dx=$ __________.

Q.17 The length of the curve $y=\sqrt{4-x^2}$ from $x=-\sqrt{2}$ to $x=\sqrt{2}$ is equal to __________.

Q.18 The system of equations

$$\begin{aligned} x+y+2z &= 2\\ 2x+3y-z &= 5\\ 4x+7y+cz &= 6 \end{aligned}$$

does NOT have a solution. Then, the value of c must be equal to __________.

Q.19 Let $y(x)$ be a solution to the differential equation

$$y''-2y'+y=0,\quad y(0)=1 \text{ and } \quad y'(0)=1.$$

Then $\lim_{x\to-\infty} y(x)=$ __________.

Q.20 The area of the region in the first quadrant enclosed by the curves $y=0$, $y=x$ and $y=\frac{2}{x}-1$ is equal to __________.

END OF THE QUESTION PAPER